Homework Assignment VII

Physics 105.2, Instructor: Petr Hořava

This assingment was originally scheduled to be due on Friday, March 21. However, in response to popular demand, the due date has now been officially changed to **Tuesday, April 1, 12 noon**.

This week we started by deriving the general relation between conservation laws and continuous symmetries (known as Noether's theorem), which is still a part of the Lagrangian formalism. Then we started moving from the Lagrangian formulation to the Hamiltonian formulation of mechanics.

The required reading for this week consists of the entire Chapter 5 of Hand-Finch, with the exception of Section 5.8, but including the two appendices. Actually, since the mid-term exam is coming up on March 20, it might be a good idea to review the entire material in Chapters 1 – 4 as well.

This week's problems are:

1. *(A new constant of the motion for the Kepler problem):*
   Hand-Finch, Problem 2(a) and (b) from Chapter 5.
2. *(Hamiltonian formulation of a simple system):*
   Consider a particle of mass m sliding under the action of gravity and without friction on a wire shaped into a parabola defined by $y = \frac{1}{2}(x - x_0)^2 - y_0$, where x, y are two Cartesian coordinates, and x_0, y_0 are some fixed constants.
   (a) Express the Lagrangian $L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) - mgy$ as a function of the independent degree of freedom x.
   (b) Identify the canonical momentum p associated with x. (c) Is the Lagrangian regular or singular? (The very important concept of regular versus singular Lagrangians was discussed in class :-)
3. *(continuation of the previous problem):*
   For the system considered in Problem 2,
   (a) find the Hamiltonian H as a function of p, x.
   (b) using this expression for H, write down Hamilton's equations of motion for this system.
4. *(Motion along a spiral):*
   Hand-Finch, Problem 4 of Chapter 5.
5. *(A familiar system expressed in momentum space):*
   Hand-Finch, Problem 16 of Chapter 5. (See Section 5.7 of Hand-Finch for the definition of the "momentum space Lagrangian.")